

## Molecular Energy Levels

[Useful conversion ratio for rotational energy levels:  $h/8\pi^2c = 16.8576 \text{ amu } \text{Å}^2 \text{ cm}^{-1}$ ]

Q 1.1 A sample of nitrogen gas is confined to a cubic volume 10 cm x 10 cm x 10 cm at 300 K. Write down the expression for the translational energy levels of a nitrogen molecule, using the three quantum numbers  $n_x$ ,  $n_y$  and  $n_z$ . Calculate the separation between the lowest two translational levels taking  $m(\text{N}) = 14.0$ . Given that  $k_B T$  at room temperature is about  $208 \text{ cm}^{-1}$  ( or  $0.414 \times 10^{-20} \text{ J molecule}^{-1}$ ), deduce what you can about the quantised motion of the nitrogen molecules.

Q 1.2 Some consecutive rotational energy levels of  $\text{H}^{35}\text{Cl}$  in the  $v = 0$  level occur at 125.201, 208.584, 312.716 and  $437.534 \text{ cm}^{-1}$ . Identify the  $J$  value for each level; hence deduce the moment of inertia and bond length of the molecule. What is the degeneracy of each level and to what does this degeneracy correspond physically?  
[ $m(^{35}\text{Cl}) = 34.969 \text{ a.u.}$ ,  $m(^1\text{H}) = 1.0078 \text{ a.u.}$ ].

Q 2.1 The vibrational levels of the diatomic molecule sodium iodide (NaI) lie at the following wavenumbers: 142.8, 427.3, 710.3,  $991.8 \text{ cm}^{-1}$ .

Deduce the values for the constants  $\omega_e$  and  $x_e\omega_e$  and the zero point energy.

Q 2.2 The molecules  $\text{O}_2$  and  $\text{N}_2$  have harmonic vibrational wavenumbers of 1580 and  $2359 \text{ cm}^{-1}$ , respectively. Calculate the bond force constants for these two molecules and comment on their respective values.  
[ $m(^{16}\text{O}) = 15.995 \text{ a.u.}$ ,  $m(^{14}\text{N}) = 14.003 \text{ a.u.}$ ].

Q 2.3 The vibrational parameters for  $\text{H}^{35}\text{Cl}$  are  $\omega_e = 2990.95 \text{ cm}^{-1}$  and  $\omega_e x_e = 52.819 \text{ cm}^{-1}$ . Calculate the  $^{35}\text{Cl}$  to  $^{37}\text{Cl}$  isotope shift for the  $v=1-0$  vibrational interval for both HCl and DCl. Can you explain why one is larger than the other?

Estimate the dissociation energies  $D_0$  for  $\text{H}^{35}\text{Cl}$  and  $\text{H}^{37}\text{Cl}$ , explaining clearly why the two values are not the same.

[ $m(^{37}\text{Cl}) = 36.966 \text{ a.u.}$ ,  $m(^2\text{H}) = 2.0141 \text{ a.u.}$  See also Q 1.2].

Q 3.1 The following are observed wavenumbers of lines in the 0-0 band of an electronic spectrum of BeO.

$J$	R( $J$ )	P( $J$ )
0	21199.8	
1	21202.9	21193.3
2	21205.7	21189.9
3	21208.5	21186.4
4	21211.1	21182.7
5	21213.6	21178.9
6	21215.6	21174.8
7		21170.7

The transitions in the R branch obey the selection rule  $\Delta J = +1$  and those in the P branch  $\Delta J = -1$ . How can you confirm the assignment of the rotational numbering? What are the  $B$  values for the two vibrational levels?

- Q 3.2 a) The molecule  $\text{Br}_2$  has a dissociation energy  $D_0 = 1.971$  eV. Its vibrational wavenumber is  $323 \text{ cm}^{-1}$ . Calculate the value of the dissociation energy  $D_e$  in  $\text{cm}^{-1}$  ( $1 \text{ eV} = 8065 \text{ cm}^{-1}$ ).
- b) A series of absorption bands is observed in the electronic spectrum of  $\text{O}_2$  in the ultraviolet region. The origins of the first three bands are at  $49363$ ,  $50\,046$  and  $50\,710 \text{ cm}^{-1}$ . Sketch an energy level diagram for the transitions (all of which originate from the  $v = 0$  level of the ground state), and estimate the dissociation energy  $D_0$  of the excited state. The dissociation energy is actually  $7194 \text{ cm}^{-1}$ . Comment.
- Q 4.1 The first excited electronic state of a molecule lies  $200 \text{ kJ mol}^{-1}$  above the ground state. Using the Boltzmann distribution law and the fact that at room temperature  $RT$  is roughly  $2.5 \text{ kJ mol}^{-1}$ , find the proportion of molecules in this excited state.
- Q 4.2 For  $\text{CO}$ , the spacing between vibrational levels 0 and 1 is around  $2100 \text{ cm}^{-1}$ . What proportion of molecules are in the state with  $v = 1$  at room temperature? Would you expect to see the transition  $v = 2 \leftarrow 1$  in the absorption spectrum?
- Q 4.3 The molecule  $\text{BF}$  has a rotational constant of  $1.52 \text{ cm}^{-1}$ . Which will be the most highly populated rotational level at  $500 \text{ K}$ ?